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CATEGORY LOGIC
Contravariant Functor
AutoLog Indexical Forcing
2/8/2025

* How to specify AutoLog Rules for a contravariant Functor
* corresponding to category theory diagrams.
* Notes regarding operators '::' vs ':'
* 1st order category indexical
* $F:X \rightarrow Y$ category mapping F from object X to object Y
* $C:cat$ C is category
* 2nd order category indexical
* $F:C \rightarrow D$ Functor F from cat C to cat D
* Consider the following autoLog theory Functor.auto
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true \Rightarrow c:cat, d:cat. // 1
true \Rightarrow x:c, y:c, fnctr::c \rightarrow d. // 2
true \Rightarrow f:x \rightarrow y, g:y \rightarrow z. // 3

C:cat, D:cat, Fnctr::C \rightarrow D, X:C \Rightarrow Fnctr(id(X))=id(Fnctr(X)). // 4

C:cat, D:cat, Fnctr::C \rightarrow D, X:C \Rightarrow Fnctr(X):D. // 5

C:cat, D:cat, Fnctr::C \rightarrow D, X:C, Y:C, Z:C, F:X \rightarrow Y, G:Y \rightarrow Z \Rightarrow
Fnctr(G \circ F)=Fnctr(F) \circ Fnctr(G). // 6 contravariant

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The AutoLog theory Functor.auto compiles as is
Next: Use rules 4,5,6 to force conclusions from 1,2,3,
using AutoLog Inference trees. This idea is intended to illustrate
how autolog indexical forcing is applicable to deriving logical
conclusions inferred from the category diagram. The intention is to
display what graphical output to expect from the AutoLog Skolem
Machine inferences for the Functor computations.

true	
	1
c:cat	
	1
d:cat	
	2
x:c	
	2
y:c	
	2
fnctr::c \rightarrow d	
	3

$f:x \rightarrow y$	
	3
$g:y \rightarrow z$	
	4
$\text{fnctr}(\text{id}(x)) = \text{id}(\text{fnctr}(x))$	
	5
$\text{fnctr}(x):d$	
	5
$\text{fnctr}(y):d$	
	6
$\text{fnctr}(g \circ f) = \text{fnctr}(f) \circ \text{fnctr}(g)$	contravariant

So far it appears that the Functor related rules for indexical forcing generally work as intended. Carefully presenting these autolog inferences as proof branches are fairly easy to follow. Notice that rules 4,5, and 6 are applied to inference branch facts generated earlier in the tree by applying rules 1,2,and 3 first.

Notice that this contravariant Functor version simply modifies rule 6 from the covariant version!

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